HOMOTOPY PERTURBATION METHOD FOR SOLVING HEAT TRANSFER ANALYSIS FOR SQUEEZING FLOW OF NANOFLUID IN PARALLEL DISKS.

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Abstract: Heat transfer analysis for the magneto-hydro-dynamics **(MHD)** Squeezing flow of a viscous incompressible fluid between parallel manifolds is considered. The upper manifold is movable in upward and downward directions while the lower manifold is fixed but permeable. Variable similarity transformation were used to convert the conservation law equations into a system of nonlinear ordinary differential equations. The resulting system of nonlinear ordinary differential equation were solved by using Homotopy Perturbation Method **(HPM)**. Influence of flow parameters is discussed and numerical solution is sought using Finite Difference Method **(FDM)**. A convergent solution is obtained just after few number of iterations.

Indexed Terms: Squeezing flow, magneto-hydro-dynamics **(MHD)**, Homotopy Perturbation Method **(HPM)**, Parallel manifolds, Finite Difference Method **(FDM)**.

1. Introduction

Heat transfer in rapidly moving engines and machines with lubricants inside has been an active field of research. For safe and consistent working of such machines it is necessary to study heat transfer in these systems. Several attempts are reported in this regard after the pioneer work done by Stefan [1]. Two dimensional **MHD** squeezing flow between parallel plates has been examined by Siddiqui et al. [2]. For parallel disk similar problem has been discussed by Domairy and Aziz [3]. Both used homotopy perturbation method **(HPM)** to determine the solution.

He. [4] did propose and applied the Homotopy Perturbation Method (HPM) technique to the Problems. Also He. [5] used HPM For Seapage flow withy. They used **HAM** to solve the resulting nonlinear system of ordinary differential equations.

Motivated by the preceding work here we present heat transfer analysis for the **MHD** squeezing flow between parallel disks. Well known homotopy perturbation method **(HPM)** has been employed to solve system of highly nonlinear differential equations that govern the flow. **HPM** is a strong analytical technique and has been employed by several researchers in recent times to study different type of problems [7–20]. The main positive features of this technique is its simplicity, selection of initial approximation, compatibility with the nonlinearity of physical problems of diversified complex nature, minimal application of integral operator and rapid convergence [10].

Numerical solution is also sought to check the validity of analytical solution. A detailed comparison between purely analytical solution obtained by HPM and the numerical solution obtained by employing FDM method is presented. It is evident from this article that the HPM provides excellent results with less amount of laborious computational work.

2. Mathematical formulation

MHD flow of a viscous incompressible fluid is taken into consideration through a system consisting of two parallel infinite disks distance $h(t) = H(1-at)^{1/2}$ apart. Magnetic field proportional to $B_0(1-at)^{1/2}$ is applied normal to the disks. It is assumed that there is no induced magnetic field. T_W and T_h represent the constant temperatures at $z = 0$ and $z = h(t)$ respectively. Upper disk at $z = h(t)$ is moving with velocity $\frac{aH(1-at)^{-1/2}}{2}$ toward or away from the static lower but permeable disk at $z = 0$ as shown in Fig. 1. We have chosen the cylindrical coordinates system (r, ϕ, z). Rotational symmetry of the flow $(\partial/\partial\phi = 0)$ allows us to take azimuthal component v of the velocity $V = (u, v, w)$ equal to zero. As a result, the governing equation for unsteady two-dimensional flow and heat transfer of a viscous fluid can be written as [6]

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial \hat{p}}{\partial r} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \frac{\sigma}{\rho}B^2(t)u, \quad (2)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial \hat{p}}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial r^2}+\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right), \quad (3)$$

$$C_P\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)=\frac{K_0}{\rho}\left(\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}+\frac{1}{r}\frac{\partial T}{\partial r}-\frac{u}{r^2}\right) +v\left\{2\frac{u^2}{r^2}+\left(\frac{\partial u}{\partial z}\right)^2+2\left(\frac{\partial w}{\partial z}\right)^2+ 2\left(\frac{\partial u}{\partial r}\right)^2+2\left(\frac{\partial w}{\partial r}\right)^2+2\frac{\partial u}{\partial z}\frac{\partial w}{\partial r}\right\}, \quad (4)$$

Auxiliary conditions are [5]

$$u=0,\quad w=\frac{dh}{dt}\quad at\ z=h(t)$$

$$u=0,\quad w=-w_0\quad at\ z=0. \quad (5)$$

$$T=T_w\quad at\ z=0$$

$$u=T_h\ \ at\ z=h(t). \quad (6)$$

u and w here are the velocity components in r and z directions respectively, μ is dynamic viscosity, $\hat{p}$ is the pressure and ρ is the density. Further T denotes temperature, K_0 is the thermal conductivity, C_P is the specific heat, v is the kinematic viscosity and w_0 is suction/injection velocity.

Using the following transformations [5]

$$u=\frac{ar}{2(1-at)}f'(\eta),\quad w=-\frac{aH}{\sqrt{1-at}}f'(\eta),$$

$$B(t)=\frac{B_0}{\sqrt{1-at}},\qquad \eta=\frac{aH}{\sqrt{1-at}},\qquad \theta=\frac{T-T_h}{T_w-T_h}, \quad (7)$$

Into Eqs. (2)-(4) and eliminating pressure terms from the resulting equations, we obtain

$$f^{iv}-S(\eta f'''+3f''-2ff''')-M^2f''=0, \quad (8)$$

$$\theta''-S\,Pr(2f\theta'-\eta\theta')-\Pr\ Ec(f''^2+12\delta^2f'^2)=0, \quad (9)$$

With the associated conditions

$$f(0)=A,\quad f'(0)=0,\qquad \theta(0)=1,$$

(10)

$$f(1)=\frac{1}{2},\quad f'(1)=0,\qquad \theta(1)=0,$$

Where S denotes the squeeze number, A is suction/injection parameter, M is Hartman number, Pr Prandtl number, Ec modified Eckert number, and δ denotes the dimensionless length defined as

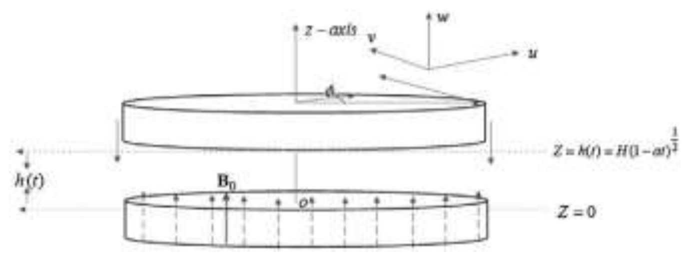
$$S=\frac{aH^2}{2v},\quad M^2=\frac{aB_0^2H^2}{v},\quad Pr=\frac{\mu C_p}{K_0},$$

$$Ec=\frac{1}{C_p(T_w-T_h)}\left(\frac{ar}{2(1-at)}\right)^2,\quad \delta^2=\frac{H^2(1-at)}{r^2} \quad (11)$$

Skin friction coefficient and the Nusselt number are defined in terms of variables (7) as

$$\frac{H^2}{r^2}Re_rC_{fr}=f''(1),\quad (1-at)^{1/2}Nu=-\theta'(1), \quad (14)$$

$$Re_r=\frac{raH(1-at)^{1/2}}{2v}. \quad (15)$$



3. Homotopy Perturbation Method (HPM)

The basic idea embodied in the HPM and a brief summary of the method can be found in [4-7]. Recent developments of the HPM can be found in series of papers by pioneering researchers such as He[21-23] and Yildirim[24-30]. Papers that bear close relevance to the present work are Mahmood et al.[31], Mehmood and Ali[32], and Z.Z. Ganji and D.D. Ganji[33], Mustapha R.A. and Salau A.M.[35],[36] and [39].

4. HPM Solution Scheme

In this section, the solution to the system of non-linear differential equations (8 – 9) subject to the boundary condition (10) is obtained via HPM.

According to HPM, we can construct an homotopy for (8,9) as follows

$$H(f,p)=(1-p)\left(f^{iv}\right)+p(f^{iv}-S(\eta f'''+3f''-2ff''')-M^2f'' \quad (16)$$

$$H(\theta,p)=(1-p)(\theta'')+p(\theta''-S\,Pr(2f\theta'-\eta\theta')-\Pr\ Ec(f''^2+12\delta^2f'^2)) \quad (17)$$

Where primes denote differentiation with respect to η to third degree and f^{iv} is representing f''''.

We are going to consider a three term-solution for f and θ in the infinite series solution which can be seen below as follows

$$f=f_0+pf_1+p^2f_2 \quad (18a)$$

$$\theta=\theta_0+p\theta_1+p^2\theta_2 \quad (18b)$$

We substitute 18a – 18b into (16) and (17) and we do some algebraic manipulation to obtain the below set of equations:

$$p^0: f_0'''=0\ ,$$

$$p^1: -m^2f_0''+2Sf_0f_0'''-S\eta f_0'''-3Sf_0''+f_1^{iv}=0$$

$$p^2: -m^2f_1''+2Sf_1f_0'''+2Sf_0f_1'''-S\eta f_1'''-3Sf_1''+f_2^{iv}=0\ , \quad (19)$$

Which is associated with the below initial conditions

$$f_0(0) = A\,, f_0'(0) = 0\,, f_0(1) = \tfrac{1}{2}\,, f_0'(1) = 0$$
$$f_1(0) = A\,, f_1'(0) = 0\,, f_1(1) = \tfrac{1}{2}\,, f_1'(1) = 0 \quad (20)$$
$$f_2(0) = A\,, f_2'(0) = 0\,, f_2(1) = \tfrac{1}{2}\,, f_2'(1) = 0$$

$$p^0: \theta_0'' = 0\,,$$
$$p^1: -12PrEc\delta^2(f_0')^2 - PrEc(f_0'')^2 + 2PrSf_0\theta_0' -PrS\eta\theta_0' + \theta_1'' = 0 \quad (21)$$
$$p^2: -24PrEc\delta^2 f_0'f_1' - 2PrEcf_0''f_1'' + 2PrSf_1\theta_0' +2PrSf_0\theta_1' - PrS\eta\theta_1' + \theta_2'' = 0$$

Which is associated with the below initial conditions

$$f_0(0) = A\,, f_0'(0) = 0, \theta_0(0) = 1\ , f_0(1) = \tfrac{1}{2}\,, f_0'(1) = 0, \theta_0(1) = 0$$
$$f_1(0) = A\,, f_1'(0) = 0, \theta_1(0) = 1\ , f_1(1) = \frac{1}{2}\,, f_1'(1) = 0, \theta_1(1) = 0$$
$$f_2(0) = A\,, f_2'(0) = 0, \theta_2(0) = 1\ , f_2(1) = \frac{1}{2}\,, f_2'(1) = 0, \theta_2(1) = 0$$
(22)

We obtained an analytical solution using the symbolic algebra package Maple16 to solve (19) and (21) with the associated boundary conditions (20) and (22)

$$f_0(\eta) = \frac{1}{6}(-6 + 12A)\eta^3 + \frac{1}{2}(3 - 6A)\eta^2 + A$$

And

$$\theta_0(\eta) = -\eta + 1$$

The above are the first term of the series solutions

5. Results and Discussion

In this section, we present the graphical solution of the afore mentioned problem to show the effect of the pertinent parameters on the fluid flow. Which validates the HPM results given by the series solution (18a) and (18b), the three term series solution is compared with those of numerical solutions obtained via Finite difference method (FDM) for fixed values of the parameters $S, A, \delta, Ec, \Pr\ and\ M$, as shown in table 1. The table shows that the solution obtained from HPM are consistent and in good agreement with the FDM solutions.

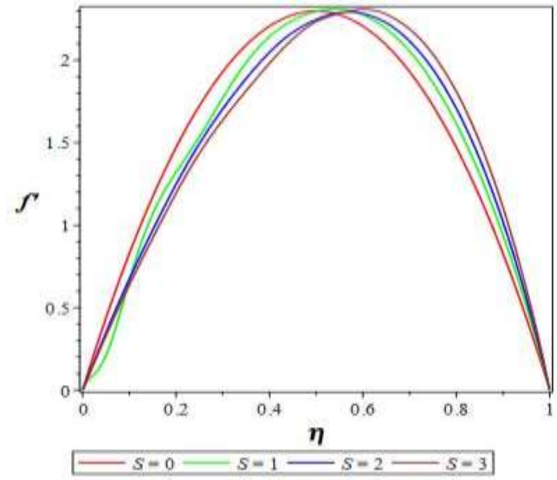


Figure 1: influence of squeezing parameter S on $f'(\eta)$ when $M = 0.1, = -1.0$.

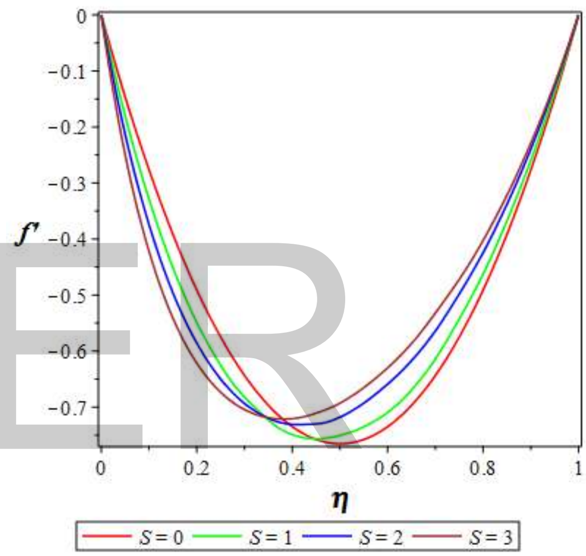


Figure 2: influence of squeezing parameter S on $f'(\eta)$ when $M = 0.5, = 1.0$.

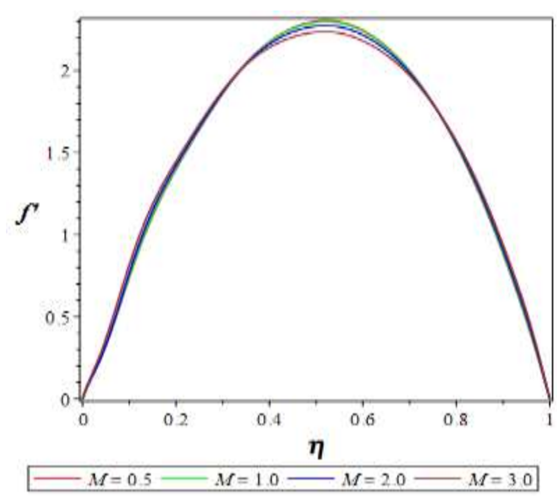


Figure 3: influence of Hartman number (M) on $f'(\eta)$ when $S = 0.5, = -1.0$.

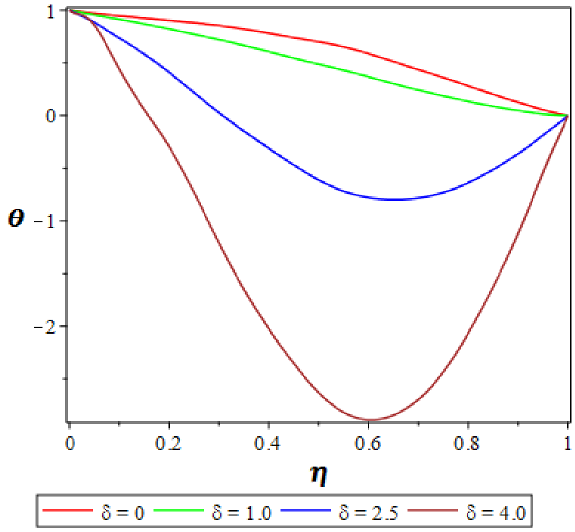


Figure 4: influence of (δ) on $\theta(\eta)$ when $S = -0.5$, $Pr = Ec = 0.1. A = 1$

	$f'(\eta)$			$\theta(\eta)$		
η	FDM	HPM	Error	FDM	HPM	Error
0	0	0	0	1	1	0
0.2	0.384801	0.38400	8×10^{-4}	0.793392	0.8000	6.608×10^{-3}
0.4	0.575554	0.57600	4.5×10^{-4}	0.592274	0.6000	7.62×10^{-3}
0.6	0.575175	0.57600	8.25×10^{-4}	0.392257	0.4000	7.74×10^{-3}
0.8	0.384040	0.38400	-4×10^{-5}	0.193382	0.2000	6.618×10^{-3}
1	0	0	0	0	0	0

Table 1.0 Comparison of Numerical and HPM solutions for diverging channel for $\delta = 0.1, A = 0.1, S = 0.1, M = 0.2, Pr = 0.3, Ec = 0.2$

Conclusion

The semi-analytical method employed to derive approximate analytical solutions for velocity and temperature profile in magneto-hydrodynamic (MHD) Squeeze flow between two parallel disks, has been found to converge very faster for the first term in the power series solution. The comparison of this method (HPM) with direct numerical solutions generated by the symbolic algebra package Maple16 which uses FDM for solving non-linear boundary value problems. The comparison of HPM and FDM in this present work are highly accurate.

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